CHAPTER 6

The Law of Absorption

The law of absorption is another very powerful tool for use in simplifying expressions. Contrary to most "laws of nature," this law illustrates a case of the "little 'un gobbling the big 'un."

An expression like A + AB can be simplified by factoring out A in accordance with the distributive law. The expression could be written as A·1 + AB. Thus, factoring out A,

$$A + AB = A(1 + B) = A$$

and since 1 + B reduces to 1 according to the law of union, the expression becomes A·1, which equals A.

Anytime a variable, group, or expression is ORed with a larger ANDed group or expression that contains the smaller variable, group, or expression, the smaller one *absorbs* the larger one.

6-1. Simplify the following expressions as illustrated above and show your work.

(a) D + DE
(b) K + KL + KM

(c) VW + W + WX
(d) SR + QRS + RSTV

(a) D + DE = D(1 + E)
= D·1
= D

(b) K + KL + KM = K(1 + L + M)
= K·1
= K

(c) VW + W + WX = W(V + 1 + X)
= W·1
= W

(d) SR + QRS + RSTV = RS(1 + Q + TV)
= RS·1
= RS

6-2. The law of absorption is:

A + AB = A   A(A + B) = A

You have just simplified expressions like A + AB. Now use the distributive law to prove that A(A + B) = A + AB.

$$A(A + B) = AA + AB = A + AB = A$$

6-3. Since A(A + B) = A + AB, expressions like A(A + B) may also be simplified to a form like A.

Simplify the expressions below and show your work.

(a) R(S + T + R)
(b) (XY + WZ + V)WZ

(a) R(S + T + R) = RS + RT + RR
= RS + RT + R
= R(S + T + 1)
= R·1
= R

(b) $(XY + WZ + V)WZ = WZXY + WZWZ + WZV$

$= WZXY + WZ + WZV$

$= WZ(XY + 1 + V)$

$= WZ \cdot 1$

$= WZ$

6-4. State the two parts of the law of absorption, and use truth tables to prove their validity.

$A + AB = A$

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

$A(A + B) = A$

A	B	A + B	A(A + B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

6-5. You have seen how to simplify expressions like $A + AB$ and $A(A + B)$ algebraically. Actually, the law of absorption is very simple. You might try plotting it on a truth table, but when we say

$$A + AB = A$$

we are simply saying what is obvious. If $A = 0$, AB will equal 0, and the output will be 0 (equal to A) at that time. If $A = 1$, the output will be 1 regardless of the value of B, so again, the output equals A, and A is seen to exercise complete control in this expression. Simplify the following expressions using the law of absorption.

(a) $ABC + AB$
(b) $XY + XYZ + WXYZ$
(c) $RS + QRS + S$

(a) AB (b) XY (c) S

6-6. Try these and remember two things about the ANDed terms:

1. The smaller term will always absorb the larger term.
2. The larger term must contain the smaller term in order for the smaller term to absorb it.

(a) $A + BC + ABC$
(b) $ST + VW + RST$
(c) $TUV + XY + Y$

(a) $A + BC$ (b) $ST + VW$
(c) $TUV + Y$

6-7. If an expression appears in the form $F(E + F + G)$, convert it to

$$FE + FF + FG = FE + F + FG$$

and by the law of absorption it becomes what?

F

6-8. Simplify $(PQ + R + ST)TS$.

$(PQ + R + ST)TS = PQST + RST + STST$

$= PQST + RST + ST$

$= ST$

6-9. Simplify the following expressions.

(a) $ABC + CB$
(b) $\overline{\overline{D}}\,\overline{\overline{D}E}$
(c) $Y(W + X + \overline{\overline{Y} + \overline{Z}})Z$
(d) $EG(\overline{BG}H + HI + GE)$
(e) $\overline{A}B + DCFE + ED + AED\overline{F}$

(a) BC (b) D (c) YZ (d) EG
(e) $\overline{A}B + ED$

6-10. Simplify the following expressions.

(a) JKL + J
(b) (BE + C + F)C
(c) (RS + TV)RS
(d) $MNP + QR + \overline{\overline{M} + \overline{N}}$
(e) (ST + W)VW

(a) J (b) C (c) RS (d) MN + QR
(e) VW

6-11. To apply the law of absorption, the variables or terms must not be in different sets of parentheses. For example, in (ABC + AB + D) (A + B) you could not use A + B to eliminate ABC or AB because A + B is within a different set of parentheses. However, you can use AB to absorb ABC, and the expression then becomes (AB + D) (A + B).

To simplify the following expressions will require the use of most of the laws you have learned so far.

(a) $(M\bar{J}K + G + K + G\bar{G}K)(G + KH + LG + K)$
(b) $(R + \bar{S})TV + \bar{S} + R$
(c) WXYZ + ZWY + VZX + VYWZ + VYZX
(d) $(CB\bar{\bar{\bar{C}}} + B\bar{\bar{\bar{D}}}E) + (\overline{\overline{ABE + C}} + \overline{\overline{BE}})$
(e) $\overline{\overline{ML}H\bar{J}KH + \bar{J}(H + K)}$
(f) DFGE + ED + CDEFG + ECF + CFG + GEDC + DFDG + CFE + DFG

(a) $(M\bar{J}K + G + K + G\bar{G}K)(G + KH + LG + K)$
$= (G + K)(G + K)$
$= G + K$

(b) $(R + \bar{S})TV + \bar{S} + R = TVR + TV\bar{S} + \bar{S} + R$
$= \bar{S} + R$

(c) For an expression such as this, look for the shortest groups—in this case ZWY and VZX. Use these to absorb other groups containing them.

WXYZ + ZWY + VZX + VYWZ + YVZX
= WYZ + VXZ

(d) $(CB\bar{\bar{\bar{C}}} + B\bar{\bar{\bar{D}}}E) + (\overline{\overline{ABE + C}} + \overline{\overline{BE}})$
$= CBC + B\bar{\bar{D}}E + ABE + C + BE$
$= C + BE$

(e) $\overline{\overline{ML}H\bar{J}KH + \bar{J}(H + K)}$
$= (ML + \bar{\bar{H}} + J + \bar{\bar{K}} + \bar{H})(J + \bar{\bar{H}}\bar{\bar{K}})$
$= (ML + \bar{\bar{H}} + J + \bar{\bar{K}})(J + \bar{\bar{H}}\bar{\bar{K}})$
$= J + (ML + \bar{\bar{H}} + \bar{\bar{K}})\bar{\bar{H}}\bar{\bar{K}}$
$= J + \bar{\bar{H}}\bar{\bar{K}}ML + \bar{\bar{H}}\bar{\bar{K}}\bar{\bar{H}} + \bar{\bar{H}}\bar{\bar{K}}\bar{\bar{K}}$
$= J + \bar{\bar{H}}\bar{\bar{K}}ML + \bar{\bar{H}}\bar{\bar{K}} + \bar{\bar{H}}\bar{\bar{K}}$
$= J + \bar{\bar{H}}\bar{\bar{K}}$

(f) The shortest group is ED, so absorb other groups containing it. Then we have

ED + ECF + CFG + DFDG + CFE + DFG
= ED + ECF + CFG + DFG + CFE + DFG
= ED + ECF + CFG + DFG + CFE
= ED + ECF + CFG + DFG

THE COMMON IDENTITIES

6-12. The common identities are *not* basic laws, but are derived from the laws of Boolean algebra. They are:

$$A(\bar{A} + B) = AB$$

$$A + \bar{A}B = A + B$$

needs distributive

Simplify $A(\bar{A} + B)$ and $A + \bar{A}B$ to show that these identities are true. Show the steps required. Also, show the validity of the identities by truth tables.

$A(\bar{A} + B) = A\bar{A} + AB$
$= 0 + AB$
$= AB$

$A + \bar{A}B = (A + \bar{A})(A + B)$
$= 1 \cdot (A + B)$
$= A + B$

or $A + B = A + B(A + \bar{A})$
$= A + BA + \bar{A}B$
$A + B = A + \bar{A}B$

$A + \overline{A}B = A + B$

A	$\overline{A}$	B	$\overline{A}B$	$A + \overline{A}B$	A + B
0	1	0	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1

$A(\overline{A} + B) = AB$

A	$\overline{A}$	B	$\overline{A} + B$	$A(\overline{A} + B)$	AB
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	1

6-13. Simplify the following expressions using the laws you have learned. Show your work, and you should begin to recognize the two forms of the common identities from this.

(a) $\overline{B}(E + B)$
(b) $K + J\overline{K}$
(c) $AB(C + \overline{AB})$
(d) $VRS + T\overline{SRV}$
(e) $(XY + Z)WT + \overline{Z + XY}$

(a) $\overline{B}(E + B) = \overline{B}E + \overline{B}B$
$= \overline{B}E + 0$
$= \overline{B}E$

(b) $K + J\overline{K} = (K + J)(K + \overline{K})$
$= (K + J)\cdot 1$
$= K + J$

(c) $AB(C + \overline{AB}) = ABC + AB\overline{AB}$
$= ABC + 0$
$= ABC$

(d) $VRS + T\overline{SRV} = (VRS + T)(VRS + \overline{SRV})$
$= (VRS + T)\cdot 1$
$= VRS + T$

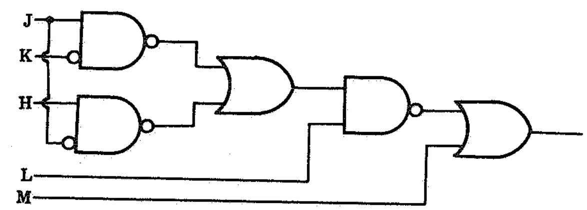
(e) $(XY + Z)WT + \overline{Z + XY} = \overline{Z}\overline{X} + \overline{Z}\overline{Y} + WT$

6-14. Simplify the following expressions.

(a) $[CD + F(\overline{B}\overline{E}) + (\overline{E + B})F + DC]G$
(b) $FG + EB\overline{B} + \overline{\overline{GF}}$
(c) $P[KJ + L(N + M) + \overline{JK}]$
(d) $B + C + BD\overline{B}D$
(e) $(XX + \overline{Y}Y)Z$
(f) $(\overline{S} + S)RST$

(g) $(A + \overline{B} + C + \overline{A} + B)(\overline{C} + \overline{A})$
(h) $RSTU + RSTV + TVWSXR$
(i) $JK(LMN + \overline{J}\overline{K}PQ)(K + P)$
(j) $\overline{XY} + \overline{X + Y}$
(k) $A + B + (C + \overline{B})(E + F)$
(l) $AC + C + (R + \overline{R})S$
(m) $\overline{WX}\overline{Y}Z + XW$
(n) $(B + C)(\overline{D} + B)(\overline{C} + B)$
(o) For the diagram shown, perform three steps.

1. Determine the output expression.
2. Simplify the expression.
3. Diagram the simplified expression.



Only the major steps will be shown to indicate the method of simplification.

(a) $(CD + 1 + DC)G = 1\cdot G = G$
(b) $FG + 0 + GF = FG$
(c) $P[L(N + M) + 1] = P\cdot 1 = P$
(d) $B + C + 0 = B + C$
(e) $(X + 0)Z = XZ$
(f) $1\cdot RST = RST$
(g) $1(\overline{C} + \overline{A}) = \overline{C} + \overline{A}$
(h) $RST(U + V + VWX) = RST(U + V)$
$= RSTU + RSTV$
(i) $(JKLMN + JK\overline{J}\overline{K}PQ)(K + P)$
$= (JKLMN + 0)(K + P)$
$= JKLMN(K + P)$
$= JKLMNK + JKLMNP$
$= JKLMN + JKLMNP$
$= JKLMN$

(j) $\bar{X} + \bar{Y} + \bar{X}\bar{Y} = \bar{X} + \bar{Y}$

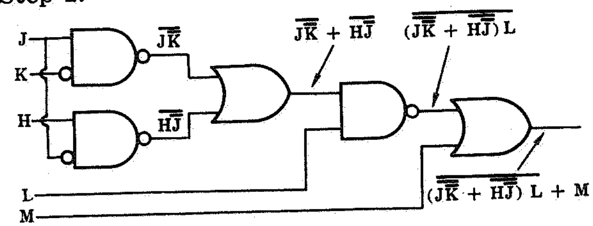
(k) $(A + B + C + \bar{B})(A + B + E + F)$
$= 1(A + B + E + F)$
$= A + B + E + F$

(l) $AC + C + (1 \cdot S) = AC + C + S$
$= C + S$

(m) $(XW + \overline{XW})(XW + \bar{Y}Z) = 1(XW + \bar{Y}Z)$
$= XW + \bar{Y}Z$

(n) $B + C\bar{D}\bar{C} = B + 0$
$= B$

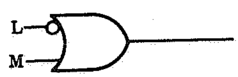
(o) Step 1.



Step 2.

$$\overline{(\overline{J\bar{K}} + \overline{H\bar{J}})L} + M = J\bar{K}H\bar{J} + \bar{L} + M$$
$$= 0 + \bar{L} + M$$
$$= \bar{L} + M$$

Step 3.



SUMMARY

1. The law of absorption is:

$$A + AB = A \qquad A(A + B) = A$$

2. Any expression of a type like $A + AB$ or $A(A + B)$ may be simplified algebraically as follows:

$$A(A + B) = AA + AB$$
$$= A + AB$$
$$= A(1 + B)$$
$$= A \cdot 1$$
$$= A$$

3. The simplest way to use the law of absorption is to convert expressions in the form $A(A + B)$ to the form $A + AB$. Then eliminate all ANDed larger groups containing the smaller ANDed groups.
4. The common identities are:

$$A(\bar{A} + B) = AB \qquad A + \bar{A}B = A + B$$

5. The common identities are not basic laws of Boolean algebra. They are derived from the laws and are useful for simplifying expressions rapidly.
6. Review all 15 of the items in 6-14 to prepare yourself for the study of Veitch diagrams which follow.