Boolean Lecture Notes #1

- covers Boolean HW #1

RELATING DIGITAL LOGIC CIRCUITS AND BOOLEAN EQUATIONS

1. Boolean algebra is a simplified mathematical system used to deal with binary or two value functions. It permits us to express all of the various logic functions, both simple and complex, in a convenient mathematical format. This system gives us a method of understanding and designing digital logic circuits.

   In Boolean algebra, logic functions are expressed ________________.

2. (mathematically) The mathematical expression of logic functions permits a convenient means of analyzing and expressing operations in digital circuits. It also aids greatly in design. The proper application of Boolean algebra usually results in the simplest, least expensive and most efficient logic circuit design.

   One of the most beneficial applications of Boolean algebra is in ________________.

3. (design) Most digital equipment in use today is made with integrated circuits. Boolean algebra is used in designing these devices. The applications of integrated circuits in the design of modern electronic equipment, also involves Boolean algebra, but only to a lesser extent. At one time, the engineer or technician designing a digital system had to design not only the logic functions but also the circuits to implement them. Boolean algebra was his primary design tool.

   The modern digital designer does not need Boolean algebra today as much as he did in previous years because of the availability of ________________ ________________.

4. (integrated circuits) The engineer and technician using and designing digital circuits, finds Boolean algebra most valuable in expressing and analyzing logic circuits. His design job is basically that of choosing and using existing integrated circuits to implement the functions required by the application. Occasionally, Boolean algebra will be used to minimize a function and achieve an efficient design.

   However the greatest benefit of Boolean Algebra to the modern designer is in ________________ and ________________ digital logic operations.

5. (analyzing or expressing) A Boolean expression is an equation that expresses the output of a logic circuit in terms of its inputs. You were introduced to Boolean expressions when you studied basic logic gates. The binary inputs and outputs were expressed as letters of the alphabet, alpha-numeric combinations, abbreviations or short words called mnemonics. For example, in the AND gate in Figure 5-1, the inputs are A and B and the output is C.

Note that the output C is expressed in terms of the inputs. The dot between the A and B indicates the AND function. The output expression $C = A \cdot B$ is read C equals A AND B. Remember that the inputs and outputs are binary signals which may assume either the binary 0 or binary 1 state.

6. [redacted] The output of a logic circuit is a function of the states of the inputs and, of course, the special logical characteristics of the circuit itself.

While there are some simple digital control operations that can be implemented with a single logic gate, more often it is necessary to use a number of logic gates to implement the desired decision-making function. When two or more logic elements are combined, the result is known as a combinational logic circuit. The circuit usually has multiple inputs and either a single output or multiple outputs depending upon its exact function.

Using AND and OR gates together to implement a special logic function creates a type of digital circuit called a ____________ circuit.

7. (combinational) Any combination of multiple ANDs. ORs. and NOTs is called a combinational logic circuit. Such circuits are used for sophisticated decision-making functions.

There are many common combinational logic circuits used in digital equipment. They perform specific functions that tend to regularly reoccur in digital equipment. However, regardless of the type of combinational logic circuits there are two basic circuit forms. These are referred to as the sum-of-products and product-of-sums circuits. Here the term product refers to the AND function while the sum refers to the OR function.

The AND function is written in the same form as the algebraic product or the multiplication of two variables. (A and B = AB) The OR function is written as the sum of two input variables. (D OR E = D + E). The sum-of-products or product-of-sums expressions combine the AND and OR functions in a variety of ways.

Combinational logic circuits are used for ____________
____

8. (decision making) Any combination of AND and OR gates is used for logical decision making purposes.

BOOLEAN RULES

9. As we mentioned earlier, the primary benefit of Boolean Algebra to a technician or engineer today is in analyzing, understanding and concisely expressing digital logic functions. The availability of a wide variety of integrated circuits has greatly minimized the use of Boolean Algebra as a design tool. However, even with modern ICs, the designer can often benefit from the use of Boolean algebra in minimizing or implementing a function.

Boolean Algebra is the algebra of two-valued functions. Many of the ordinary rules of algebra such as factoring or expanding a function, apply to Boolean expressions. However, the binary nature of the functions greatly simplifies most of the operations. There are also numerous special rules that apply to handling binary logic functions. We will explain these rules in this section and show how they are used.

Most standard algebra rules work with Boolean expressions. True or False? ______________.

10. (True) Yes, most conventional algebra manipulations will work on binary expressions. But it is the special rules that are of the most value to the digital designer.

The Laws of Intersection, for example, apply to AND gates. The two forms of this law are stated below.

$$A \cdot (1) = A$$
$$A \cdot (0) = 0$$

Remembering that A is a binary signal that can be either binary 0 or binary 1, we can prove the validity of these expressions if we remember how an AND gate works. The first expression simply says that if we apply a binary 1 to one input of an AND gate and the signal A to the other input, the output will be A. The binary 1 input simply enables the gate so that the A input state controls the output. If A = 1, the output will be 1. If A = 0, the output will be 0.

11. The other form of the Laws of Intersection is just as easy to understand.

$$A\,(0) = 0$$

It says that if one input to an AND gate is 0 and the other is A, the output will always be 0. Remember that the only time the output of an AND gate can be 1 is when all inputs are binary 1. If one input is fixed at 0, the output will always be 0. The circuit in Figure 5-33 expresses this.

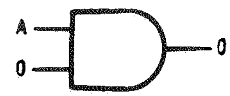

Figure 5-33

Complete the expression: $D \bullet E \bullet (0) =$ ______________.

12. $(D \bullet E \bullet (0) = 0)$ If one input to an AND gate is 0, the output will be zero regardless of the states of the other inputs. This proves that the Law of Intersection works for AND gates with more than two inputs. For example, $D \bullet E \bullet (1) = D \bullet E$. You can also equate this expression to the algebraic version that says that a function (DE) multiplied by 1 is the function.

$$DE \times 1 = DE$$

Another similar set of rules exist for OR gates. These are called the Laws of Union. Expressed algebraically they are

$$B + 1 = 1$$
$$B + 0 = B$$

13. The Laws of Tautology apply to both AND gates and OR gates. The basic rules are given below.

$$A \bullet A = A$$
$$B + B = B$$

The related logic symbols are shown in Figure 5-35.

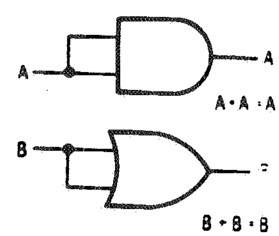

Figure 5-35

What these expressions say is that if you apply the same signal to all inputs of a logic gate, the output will be the same as the input. Again, you can prove this to yourself by looking at the truth tables for AND and OR gates.

Use the Laws of Tautology to simplify the expression $QT = JMX + JMX + F9$. ______________.

14. Another Boolean law is the Law of Complements. These are

$$A \cdot \overline{A} = 0$$
$$B + \overline{B} = 1$$

If we apply a logic signal and its complement to a logic gate, the output becomes either a binary 0 or a binary 1 depending on type of logic gate. This is illustrated in Figure 5-38.

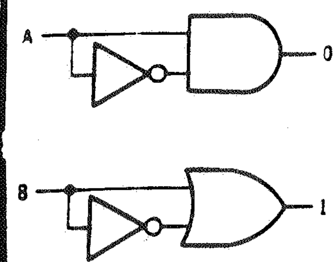

Figure 5-38

15. Now consider the Law of the Double Negative.

$$\overline{\overline{A}} = A$$

It says that the complement of the complement of A is equal to A. Or a signal that is complemented twice is the same as the original signal. You can see this from the circuit in Figure 5-39.

16. Another Boolean rule is the Law of Commutation. This is the same rule from basic algebra. The two forms of it are:

$$A \cdot B = B \cdot A$$
$$A + B = B + A$$

All it really says is that you can arrange the inputs to AND or OR gates in any order and the effect is the same. You can write the input variables in any order and they will mean the same thing.

$$W + X + Y = X + W + Y$$

Now let's give you some practice in using these rules. What are the simplifications of the expressions below?

a. $A + \overline{B} + A =$ ______________
b. $B C \overline{B} =$ ______________
c. $C + 1 + \overline{B} =$ ______________
d. $\overline{X} + Y + X =$ ______________

Work these problems using the previously discussed rules. Check your answers in the next frame.