INVERSION OF FUNCTIONS
De Morgan's THEOREM

De Morgan's theorem simply reduces this relation to an equality of two expressions by stating:

$$\overline{AB} = \overline{A} + \overline{B}$$

The other half of De Morgan's theorem states that:

$$\overline{A + B} = \overline{A}\,\overline{B}$$

We will prove this proposition in the programmed text that follows, discover how the theorem can be extended to apply to larger expressions, and find a simplified method of "De Morganizing" an expression.

4-1. Complete the following truth tables.

$\overline{A}$	$\overline{B}$	A	B	AB	$\overline{AB}$	$\overline{A} + \overline{B}$
1	1	0	0	0	1	1
1	0	0	1	0	1	1
0	1	1	0	0	1	1
0	0	1	1	1	0	0

$\overline{AB} = \overline{A} + \overline{B}$

$\overline{A}$	$\overline{B}$	A	B	A + B	$\overline{A + B}$	$\overline{A}\,\overline{B}$
1	1	0	0	0	1	1
1	0	0	1	1	0	0
0	1	1	0	1	0	0
0	0	1	1	1	0	0

$\overline{A+B} = \overline{A}\,\overline{B}$

If the question in your mind is "Why?" the answer is again rather simple: There are many times in the simplification of expressions that we cannot handle an expression which has an overbar extending over more than one variable.

80. (Simplify, minimize, or reduce) DeMorgan's theorem provides one more tool for you to use in minimizing certain types of logic expressions. In addition, it can be used to change the form of an expression for AND to OR or OR to AND.

The following procedure can be used in making the conversions.

To change a Boolean expression from one form to another, use the following procedure:

1. Change all AND (•) expressions to OR (+) expressions.
2. Complement the individual terms that were ANDed or ORed.
3. Complement the entire expression.

Let's try it out on the expression $\overline{AB}$.

1. Change AND to OR or vise versa.

$$\overline{AB} \quad \text{becomes} \quad \overline{A + B}$$

2. Complement each term.

$$\overline{A + B} \quad \text{becomes} \quad \overline{\overline{A} + \overline{B}}$$

3. Complement entire expression______

$$\overline{\overline{A} + \overline{B}} \quad \text{becomes} \quad \overline{\overline{\overline{A} + \overline{B}}} = \overline{A} + \overline{B}$$

The result $\overline{AB} = \overline{A} + \overline{B}$ is one of our DeMorgan's relationships. Now try converting the expression $\overline{A}\,\overline{B}$ using this procedure.

81. ($\bar{A}\,\bar{B} = \overline{A + B}$) (See procedure below)

1. $\bar{A}\,\bar{B}$ Change AND to OR $\bar{A} + \bar{B}$
2. $\bar{A} + \bar{B}$ Complement each term $\bar{\bar{A}} + \bar{\bar{B}} = A + B$
3. $A + B$ Complement entire expression $\overline{A + B}$

The procedure is easily remembered and can be applied to other more complex expressions. The examples below illustrate some typical applications.

$\overline{A\bar{B}}$ becomes

$\overline{A + \bar{B}}$ then

$\overline{\bar{A} + B}$ and finally

$\overline{\overline{\bar{A} + B}} = \bar{A} + B = \overline{A\,\bar{B}}.$

Note that while only two terms are shown in the basic DeMorgan's theorems, the same rules apply to logic expressions with three or more terms.

$X + Y + Z$ becomes

$X \cdot Y \cdot Z$ then

$\bar{X}\,\bar{Y}\,\bar{Z}$ and finally

$\overline{\bar{X}\,\bar{Y}\,\bar{Z}} = X + Y + Z$

Use DeMorgan's rules to change the form of the expression:

$\overline{\bar{J} + K + \bar{L}}$

$\overline{\overline{\bar{\bar{J}} \cdot \bar{K} \cdot \bar{\bar{L}}}} =$

$J \cdot \bar{K} L$

With this information in mind, practice applying De Morgan's theorem to the following expressions.

(a) $\overline{JKL}$
~~(b) $\bar{R} + \bar{S} + \bar{T} + \bar{U}$~~
~~(c) $\overline{D + E + F + G}$~~
(d) $\bar{W}\bar{X}\bar{Y}$
(e) $\overline{DEFG}$

(a) $\bar{J} + \bar{K} + \bar{L}$ (b) $\overline{RSTU}$ (c) $\bar{D}\bar{E}\bar{F}\bar{G}$
(d) $\overline{W + X + Y}$ (e) $\bar{D} + \bar{E} + \bar{F} + \bar{G}$

Here is where the concept of *natural* groups becomes important. In changing an expression from one form to the other, the original grouping *must* be retained. For this reason, an expression like $\overline{AB + C}$ becomes

$$(\overline{A} + \overline{B})\overline{C}$$

Note that the natural group AB has been retained by enclosing it in parentheses.

Keeping this and the first part of this section in mind, apply De Morgan's theorem to the following expressions.

(a) $\overline{H + JL}$

(b) $(\overline{T} + \overline{V})\overline{W}$

~~(c)~~ $\cancel{\overline{R + LM}}$

(d) $\overline{A} + \overline{B} + \overline{E}$

(e) $\cancel{(\overline{S} + \overline{T})\overline{R} + \overline{P}}$

(f) $\cancel{\overline{CAB + F + D}}$

(a) $\overline{H}\,(\overline{J} + \overline{L})$

(b) $\overline{TV + W}$

(c) $\overline{R}(\overline{L} + \overline{M})$

(d) $\overline{ABE}$

(e) $\overline{(ST + R)P}$

(f) $(\overline{C} + \overline{A} + \overline{B})\overline{F}\,\overline{D}$

Apply De Morgan's theorem to the expressions below. Remember that in an expression like A + B(C + D), B is considered to be *grouped* with (C + D) because the single variable and the expression are ANDed (written together without an operator).

(a) $\overline{W}\,\overline{X}(\overline{Y} + \overline{Z})$

~~(b)~~ $\cancel{\overline{A + BCD}}$

(c) $\overline{F}(\overline{G} + \overline{H}) + \overline{H}$

(a) $\overline{W + X + YZ}$

~~(b)~~ $\cancel{\overline{A}(\overline{B} + \overline{C} + \overline{D})}$

(c) $\overline{(F + GH)H}$

4-6. If an overbar extends over only part of an expression, only that part under the overbar should be split using De Morgan's theorem:

Operators *under* the overbar will *change.*

Operators *not under* the overbar *do not change.*

Apply De Morgan's theorem to the following expressions:

(a) $D + \overline{G(E + F) + H}$ (split)

~~(b)~~ $\cancel{A\overline{BC} + \overline{Z + T}}$ (split)

(c) $\overline{(QR + S)N} + \overline{PM}$ (split)

(d) $\overline{W(V + XYZ)}\ (A + B)$ (split)

~~(e)~~ $\cancel{(\overline{JK} + \overline{LM})(\overline{P} + \overline{N})}$ (join)

(f) $\overline{B} + \overline{C(D + E)} + \overline{F}$ (join)

(a) $D + (\overline{G} + \overline{E}\,\overline{F})\overline{H}$

(b) $A(\overline{B} + \overline{C}) + \overline{Z}\,\overline{T}$

(c) $(\overline{Q} + \overline{R})\overline{S} + \overline{N} + \overline{P} + \overline{M}$

(d) $[\overline{W} + \overline{V}(\overline{X} + \overline{Y} + \overline{Z})]\ [A + B]$

(e) $\overline{JKLM + PN}$

(f) $\overline{BCF(D + E)}$